Nemeth Spatial Arrangements

This workshop will show rules and examples for spatial arrangements from elementary through college level transcription.

Spatial Problems

- See Nemeth Code Section 185.
- There are blank lines above and below spatial problems, even when there is a page change indicator or box above or below them. A blank line is required between a running head and spatial problems that follow.
- The separation line is made up of dots 25.
- There must be one blank cell between separation lines when more than one problem is written across the page when the problems are not numbered.
- The separation line goes one cell to the right and left of the widest item in the problem.
- When spatial arrangements are identified by a number or letter, no symbol of one spatial arrangement may be less than 3 cells distant from any symbol on any line except separation lines.

Addition and Subtraction

- Plus and minus signs are placed in the cell to the left of the widest number which appears above the separation line. If they are deliberately misaligned, follow print. Section 178 Nemeth Code.

\[
\begin{array}{c}
4 \\
+ \ 3 \\
\hline
\end{array}
\]

\[
\begin{array}{c}
\downarrow \\
\downarrow \\
\downarrow \downarrow \\
\downarrow \downarrow \downarrow
\end{array}
\]

- In the problem below, even though the separation line in the problem is longer in print, it still extends only one cell to the right and left of the longest part of the problem.
- In the problems below, notice the one blank cell between the separation lines.

\[
\begin{array}{cccc}
  8 \\
+ 3 \\
\hline
11 \\
\end{array}
\]

\[
\begin{array}{cccc}
  10 \\
- 7 \\
\hline
  3 \\
\end{array}
\]

\[
\begin{array}{cccc}
  9 \\
+ 7 \\
\hline
  6 \\
\end{array}
\]

\[
\begin{array}{cccc}
  7 \\
- 3 \\
\hline
  4 \\
\end{array}
\]

\[
\begin{array}{cccc}
  9 \\
- 3 \\
\hline
  6 \\
\end{array}
\]
When an arrangement contains fractions, fraction lines must be vertically aligned, with each numerator right justified in the column for numerators and each denominator must be left justified in the column for denominator. Fraction indicators are also vertically aligned. Nemeth Code Section 178 e.

65. Add the weights.

*Estimate:*

\[ 59 + 24 + 17 + 29 + 58 = 187 \text{ tons} \]

*Exact:*

\[
\begin{align*}
58 & \frac{1}{2} = 58 \frac{12}{24} \\
23 & \frac{5}{8} = 23 \frac{15}{24} \\
16 & \frac{5}{6} = 16 \frac{20}{24} \\
29 & \frac{1}{4} = 29 \frac{6}{24} \\
+ 58 & \frac{1}{3} = 58 \frac{8}{24} \\
\hline
184 & \frac{61}{24} = 184 + 2 \frac{13}{24} = 186 \frac{13}{24}
\end{align*}
\]

The total weight is 186 \frac{13}{24} tons.
• The problem is in 1/3, with displayed material in 5/7.
• The arrangement contains mixed fractions.
• Notice the blank spaces in the 4 and 5th line of the spatial arrangement. This is to align the fraction.
• The numeric indicator is not used within a spatial problem, even after the equals.

\[ \frac{1}{3} \text{ and 5/7 weights.} \]
\[ \text{Estimate:} \]
\[ \text{Exact:} \]
\[ \text{Total weight is 120.4 tons.} \]
- The example below is aligned as shown in print.
- The numeric indicator is not used within the spatial problem, not even after the equals.
- Author’s notes shown to the right of the problem are aligned at the right in braille. If a problem won’t fit, it should be keyed with the key placed before the problem.
- Only the portions that won’t fit should be keyed. Literary numbers are used for the key.
- The example below is part of a displayed expression.

\[
\begin{align*}
-5x - 5y &= -205 \\
5x + 11y &= 307 \\
6y &= 102 \\
y &= 17
\end{align*}
\]

Multiply equation (1) by \(-5\).

Add.

Divide by 6.

---

KEY: PROVE 2L

1L: MULTIPLY EQUATION R/H BY \(-5\).

\[
\begin{align*}
-5x - 5y &= -205 \\
5x + 11y &= 307 \\
6y &= 102 \\
y &= 17
\end{align*}
\]

ADD.
Multiplication

- The multiplication cross is written in the cell immediately to the left of the first digit (multiplier) above the line. Section 179 Nemeth Code.
- The separation line goes 1 cell to the right and left of the overall width of the problem.

\[
\begin{array}{c}
493 \\
\times 2
\end{array}
\]

- The example below shows a series of problems across the page. There is one blank cell between the separation lines of the problems.

\[
\begin{array}{ccccccc}
2 & 4 & 6 & 3 & 0 \\
\times 0 & \times 0 & \times 0 & \times 0 & \times 7 \\
9 & 8 & 2 & 1 & 3 \\
\times 1 & \times 1 & \times 1 & \times 5 & \times 1
\end{array}
\]
# More Multiplication and Division Facts

Find the product or quotient.

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<tbody>
<tr>
<td>1.</td>
<td>9</td>
<td>2.</td>
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<td>$\times$ 3</td>
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<td>$\times$ 9</td>
<td>$\times$ 8</td>
<td>$\times$ 9</td>
<td>$\times$ 3</td>
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</table>

| 7. | 2 | 8. | 9 | 9. | 6 | 10. | 8 | 11. | 8 | 12. | 7 |
| $\times$ 8 | $\times$ 1 | $\times$ 9 | $\times$ 6 | $\times$ 8 | $\times$ 9 |

| 13. | 8 | 14. | 9 | 15. | 8 | 16. | 2 | 17. | 4 | 18. | 7 |
| $\times$ 7 | $\times$ 9 | $\times$ 1 | $\times$ 9 | $\times$ 8 | $\times$ 8 |

| 19. | 9 | 20. | 7 | 21. | 6 | 22. | 9 | 23. | 8 | 24. | 3 |
| $\times$ 4 | $\times$ 7 | $\times$ 7 | $\times$ 6 | $\times$ 2 | $\times$ 9 |
• The Review and Name at the top of the page are left aligned in cell 1.
• The heading for the worksheet is centered. A blank line is required before a centered heading unless it follows a page change indicator. A blank line is required after the centered heading.
• Directions are in cell 5. A blank line is required below the directions because of the spatial material that follows. Section 185, Nemeth Code.

**Review** imad right w. of
**Review** by **practice**

in _____

**MULTIPLIED **X  **DIVISION **FACTS

**PRODUCT **OR **QUOTIENT**

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Mary Denault 2012 CTEBVI Spatial Arrangements Workshop
• Comments are to the right of the problem. Since they fit on the page, they don’t have to be keyed.
• The problem is in 1/3, the displayed spatial problem is in 5/7.
• Notice the alignment with the comma in the arrangement below.

73. First multiply 9352 by 4. Then multiply 9352 by 6, making sure to line up the tens. Then multiply 9352 by 2, making sure to line up the hundreds. Then add the partial products.

\[
\begin{array}{c}
9352 \\
\times 264 \\
\hline
37408 \quad \leftarrow 4 \times 9352 \\
56112 \quad \leftarrow 6 \times 9352 \\
18704 \quad \leftarrow 2 \times 9352 \\
\hline
2468928
\end{array}
\]
Cancellation in Subtraction

- When showing cancellation in spatial subtraction problems, the numeric indicator isn’t used for the numbers written above the cancellation. Refer to Section 60 of the Nemeth Code.
- Items that are individually cancelled in print should be individually cancelled in braille.
- The problem number goes on the main line of the problem, not where the cancellation is.

Opening Cancellation ☯
Closing Cancellation ☥

\[
\begin{array}{c}
\phantom{1} \\
1 & \cancel{2} & 11 & 11 \ \\
\hline
2, & 0 & 2 & X \\
\hline
- & 5 & 7 & 6 \\
\hline
4 & 4 & 5
\end{array}
\]
Carried Numbers

- When there are carried numbers in the arrangement, the carried number indicator must be inserted. Nemeth Code Section 178 d.
- The carried number indicator must be the same length as the separation line.
- Carried number indicator ✦✦✦✦✦✦✦

*Step 4*
Add the thousands column, including the regrouped 1.

\[
\begin{array}{c}
112 \\
18 \\
708 \\
9286 \\
+ \phantom{112}636 \\
\hline
10648
\end{array}
\]
Cancellation in Fractions

- Cancelled fractions are done spatially. Refer to Section 60 of the Nemeth Code. Remember to use the numeric indicator in the portion written above the cancellation when a number stands alone. When a number begins the fraction and is not within cancellation indicators, the numeric indicator is used.

- Refer to page N5 of the Nemeth Revisions in the 2007 BANA Update. This explains why the number indicator is used for cancelled fractions, but not in numbers aligned for computation.

- The numeric indicator is not used within cancellation indicators.

- Individual units are cancelled separately.

- The problem number is in cell 1. Divisions are in 3/5. The displayed expression is in 7/9.

- If there are runovers within cancelled fractions that don’t have numbers written above the cancellation, the cancellation with runovers are centered above the fraction line.

13. a. Explain the difference in the two approaches used to simplify \(\frac{20}{28}\).

\[
\frac{1}{4\cdot 5} \text{ and } \frac{1}{6\cdot 5}
\]

b. Are the results the same?
53. Write the prime factorization of both numerator and denominator. Then divide both numerator and denominator by any common factors, and write a 1 by each factor that has been divided. Finally, multiply the remaining factors in both numerator and denominator.

\[
\frac{160}{256} = \frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 5}{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2} = \frac{5}{8}
\]
- This was part of a displayed problem. The cancelled part wouldn’t fit across the page unless it was moved over to the margin. The preference is to keep the spatial cancelled fraction on the same line and not have runovers within the fraction.
- Crossed out sections are cancelled together.

\[ a_1x = \frac{-a_1b_1c_2 + a_2b_1c_1 + a_1b_2c_1 - a_2b_1c_1}{a_1b_2 - a_2b_1} \]
25. Write $36$ as $\frac{36}{1}$ and multiply.

$$36 \cdot \frac{5}{8} \cdot \frac{9}{15} = \frac{9}{36} \cdot \frac{1}{8} \cdot \frac{3}{\frac{9}{2}} \cdot \frac{2}{1} = \frac{27}{2} = 13\frac{1}{2}$$
Synthetic Division

- In synthetic division, the numeric symbols in the synthetic dividend, synthetic product, and synthetic quotient must be aligned by place value. Section 182, Nemeth Code.
- Symbols of operation must also be aligned when present.
- Follow print for where the vertical bar appears—to the left or the right. It must be unspaced from the dividend and divisor.
- The separation line must begin directly under the vertical line at one end and go one cell beyond the overall product.
- If the synthetic divisor appears in print as boxed on both sides, ignore this in braille.

Synthetic Division Indicators

| = | Dots (456) straight line |
| PPP = | Dots (25) separation line |

Left and right synthetic division are shown below.

```
=|
  =|
  PPP =|
  =|
  =|
```
The example below is a spatial arrangement. Shape indicators are not used in spatial material, the full cell is used instead. Nemeth Code Section 58,

\[
\begin{array}{c|c|c|c|c|}
1 & 2 & 4 & -9 & \\
\end{array}
\]

**Step 1** Write the value of \( k \) and the coefficients of the dividend in order of descending exponents.

**Step 2** Write the value \( \pm k \) and coefficients as dividend in order of descending exponents.

If we write the 2 in the quotient on the bottom line, the bottom line gives both the coefficients of the quotient and the remainder. The top line now can be eliminated, and the division appears as

\[
\begin{array}{c|c|c|c|c|}
-3 & 2 & 4 & -3 & 10 \\
\hline
 & -6 & -30 & -81 \\
2 & 10 & 27 & 91 \\
\end{array}
\]

The bottom line was obtained by subtracting the middle line from the top line. If we replace the \(-3\) in the divisor with \(+3\), the signs of each number in the middle line will be reversed in the division process. Then the bottom line can be obtained by addition, and we have the final form of the synthetic division.

\[
\begin{array}{c|c|c|c|c|}
+3 & 2 & 4 & -3 & 10 \\
\hline
 & 6 & 30 & 81 \\
2 & 10 & 27 & 91 \\
\end{array}
\]

The coefficients of the dividend.

Thus,

\[
\frac{2x^3 + 4x^2 - 3x + 10}{x - 3} = 2x^2 + 10x + 27 + \frac{91}{x - 3}
\]
If we write $x^2 + x$ as a quotient on a bottom line, a bottom line gives both coefficients as quotient as remainder. As top line, etc. eliminate as division appends:

\[
\begin{array}{c|cc}
 & 1 & 1 \\
\hline
1 & 1 & 1
\end{array}
\]

As bottom line we obtain subtract by $\times$ minutes lie on top line. If we replace $x$ by $x$ divisor $+ - \div$ signs + $\times$ minutes $\times$ minute lie $\div$ replace $\div$ divisor process. Why $x$ bottom line can obtain division $x$ we has final $x$ synthetic division.
$y = a \cdot x + b$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
</tbody>
</table>

$y = 2x$
We will use synthetic division and find the remainder.

\[
\begin{array}{c|cccc}
   i & 1 & -1 & +1 & -1 \\
   \hline
   & 1 & -1-i & +1 & 0 \\
\end{array}
\]

Since the remainder is 0, \( P(i) = 0 \) and \( i \) is a zero of \( P(x) \).

Find \( P(-i) \).
Division

- See Nemeth Code Section 180. When a problem only contains a divisor and dividend, but no quotient or partial products, a spatial arrangement is not used. Example:

\[
\begin{array}{c}
(d) \ 37 \overline{0} \\
\end{array}
\]

- The example below has a quotient, and is done spatially.

\[
\begin{array}{c}
0 \\
(d) \ 129 \overline{0} \\
\end{array}
\]

- Symbols for the dividend and partial products and differences must be aligned as in print.
- The separation line begins in the column containing the division symbol and extends one cell beyond the longest part of the problem.
- The blue type in the remainder does not need to be shown in braille.

b. To divide \( P(x) \) by \( x - 1 \), we proceed as follows:

\[
\begin{array}{c}
3x^2 - 2x + 1 \\
\hline
3x^3 - 5x^2 + 3x - 10 \\
3x^3 - 3x^2 \\
\hline
-2x^2 + 3x \\
-2x^2 + 2x \\
\hline
+ \ x - 10 \\
\hline
x - 1 \\
\hline
- 9 \\
\end{array}
\]

Note that the remainder is equal to \( P(1) \).
2.B. Divide SPRX by X-1. We proceed as follows:

...
The example below is done this way for either a caret or an arrow to show movement.

Divide 337.5 by 13.5 to find the unit rate.

\[
\begin{align*}
\text{25.} \\
13.5 \text{)337.5 } \\
\text{262.5} \\
\text{150} \\
\text{135} \\
\text{15}
\end{align*}
\]

\[
\frac{337.5 \text{ miles}}{13.5} \div 13.5 = 25 \text{ miles}
\]

\[
\frac{13.5 \text{ gallons}}{13.5} \div 13.5 = 1 \text{ gallon}
\]
Since the key for expressions shown to the right won’t fit on the same braille page as the division problem, using facing pages would be helpful for the reader.

15. \[
\begin{array}{c}
\frac{3.1}{0.006} \\
\end{array}
\]

\[
\begin{array}{c}
516.666 \\
0.006 \overline{3.100000} \\
\hline
30 \\
10 \\
6 \\
40 \\
36 \\
40 \\
36 \\
40 \\
36 \\
40 \\
36 \\
4 \\
\end{array}
\]

Line up decimal points. Move decimal point in divisor and dividend three places. Write 00 in dividend. Write 0 in dividend. Stop and round answer to the nearest hundredth.

The quotient is 516.67 (rounded).
• In the example below, there are arrowed notes to the right of the problem. The arrows have been omitted.
• Numbers are aligned as shown in the problem.
Have You Seen This?

- The example below is shown in the Nemeth lesson manual, page 296. I forgot that it was covered until this came along in a textbook.
EXAMPLE W/DIVISION TA REMAINING

DIVIDE 4... BY 4... 
DIVIDE 4... INTO 4... 

... DIVIDE 4... INTO 4... 
... DIVIDE 4... INTO 4... 

ANSWER 4...
DIVIDE 4... INTO 4... 
DIVIDE 4... INTO 4... 
DIVIDE 4... INTO 4... 
DIVIDE 4... INTO 4... 
DIVIDE 4... INTO 4...
Enlarged Grouping for Notes

- The note shown to the right of the enlarged brace can be divided between lines. Runovers are indented 2 cells to the right for the note.
- The enlarged grouping symbol can be continued on extra lines if the comment is longer than the number of lines for the expression.
- In this type of arrangement, if there isn’t an enlarged brace, but the notes apply to a section, an enlarged brace can be inserted.
For Discussion

- What would you do with this?
- Is this described well in the text, or would you need to draw the arrows or make extra explanations?
For Discussion

What would you do with the problems below?

- Will the notes fit across the page with the spatial division problem?
- What are some of the issues with the second problem?
Systems of Equations

- Since this type of equation is considered to be spatial, it requires a blank line before and after it. The components should be aligned as shown in print.
- If they are referred to as systems, use this format. See Section 184 for the rules. Also see pages 316 and 317 in the lesson manual.
- There is a blank line between the directions and the system of equations that follows.
- Notice that the comma goes on the first line of the equations.

Solve each system graphically. Be sure to check your solution. If a system has an infinite number of solutions, use set-builder notation to write the solution set. If a system has no solution, state this. Where appropriate, round to the nearest hundredth.

17. \( x - y = 3, \)  \( x + y = 5 \)
18. \( x + y = 4, \)  \( x - y = 2 \)
19. \( 3x + y = 5, \)  \( x - 2y = 4 \)
20. \( 2x - y = 4, \)  \( 5x - y = 13 \)
Unified Expressions

- A unified system of equations is spatial. This requires a blank line between directions and the system of equations that follow.
- Opening enlarged grouping is used for the brace that is shown preceding each system. Refer to Nemeth Code Section 126.
- When enlarged grouping is used to unify an expression, each item begins right after the grouping symbol. Align as shown in print.

Use the graphing method to solve each system:

\[
\begin{align*}
\text{a. } & \begin{cases} 3x + y = 1 \\ -x + 2y = 9 \end{cases} \\
\text{b. } & \begin{cases} 2x - 3y = 4 \\ 4x = -4 + 6y \end{cases} \\
\text{c. } & \begin{cases} y = 4 - x \\ 2x + 2y = 8 \end{cases}
\end{align*}
\]
In the example that follows, there is room on the first line for the rest of the sentence. It goes on the top line of the system of equations.

- The system is aligned as shown in print.

Example 1

Solve the system of equations:

\[
\begin{align*}
\text{Example 1} & \\
\text{Example 1} & \\
\text{Example 1} & \\
\end{align*}
\]

\[
\begin{align*}
x^2 + y^2 & = 25 \\
2x + y & = 10
\end{align*}
\]

by graphing.
Matrices and Determinants

- Refer to Section 183 of the Nemeth Code for more information.
- In determinants or matrices each entry must be left-justified in the column that it applies to.
- One column of blank cells must be left between columns.
- The numeric indicator is used with numeric entries in a determinant or matrix, even when they are in direct contact with the opening grouping sign.
- Runovers are indented 2 cells from the column margin.
- Space can be saved by drawing the enclosure symbols.
- Fractions may be represented spatially to save space. A blank line must be left before and after when a spatial fraction is used.
- The line for this arrangement is dots 456:

\[
\begin{bmatrix}
1 & -8 & 3 \\
0 & 7 & -14 \\
\end{bmatrix} \frac{1}{7} R_2
\]  
\[
= \begin{bmatrix}
1 & -8 & 3 \\
0 & 1 & -2 \\
\end{bmatrix}
\]
A determinant of a matrix represents a single number. We obtain this value by multiplying and adding its elements in a special way. We can use the determinant of a matrix to solve a system of simultaneous equations.

For example, if we have the (square) $2 \times 2$ matrix:

$$
\begin{pmatrix}
5 & 7 \\
2 & -3
\end{pmatrix}
$$

The determinant of this matrix is written within vertical lines as follows:

$$
\begin{vmatrix}
5 & 7 \\
2 & -3
\end{vmatrix}
$$
The example below shows a unified system of equations followed by a matrix.

The equations are displayed to narrative text, and are in 3/5.

In the last section, R1 and R2 require a multipurpose indicator to show that the numbers are not subscripts to the R.

The equation \((-2)R1 + R2\) goes on the top line of the matrix.

The equations are divided before the arrow, which is a sign of comparison.

**Example 2**

Solve the following system by transforming its system matrix into row echelon form and back substituting.

\[
\begin{align*}
    x + 2y + 3z &= 4 \\
    2x - y - 2z &= 0 \\
    x - 3y - 3z &= -2
\end{align*}
\]

**Solution**

This system is represented by the system matrix

\[
\begin{bmatrix}
    1 & 2 & 3 & 4 \\
    2 & -1 & -2 & 0 \\
    1 & -3 & -3 & -2
\end{bmatrix}
\]

We will reduce the system matrix to row echelon form. We can use the 1 in the upper left corner to zero out the rest of the first column. To do so, we multiply the first row by \(-2\) and add the result to the second row. We indicate this operation with the notation \((-2)R1 + R2\)

\[
\begin{bmatrix}
    1 & 2 & 3 & 4 \\
    2 & -1 & -2 & 0 \rightarrow & 0 & -5 & -8 & -8 \\
    1 & -3 & -3 & -2
\end{bmatrix}
\]
Example 1:

Solve 3-plex system: Transpose of system matrix times system matrix equals back substitution.

\[
\begin{bmatrix}
1 & 2 & 3 \\
4 & 5 & 6 \\
7 & 8 & 9
\end{bmatrix}
\times
\begin{bmatrix}
1 & 2 & 3 \\
4 & 5 & 6 \\
7 & 8 & 9
\end{bmatrix}
= \begin{bmatrix}
1 & 2 & 3 \\
4 & 5 & 6 \\
7 & 8 & 9
\end{bmatrix}
\]

Solution:

The system is represented as a system matrix

\[
\begin{bmatrix}
1 & 2 & 3 \\
4 & 5 & 6 \\
7 & 8 & 9
\end{bmatrix}
\]

We reduce the system matrix to reduce

We use only 3 x 3 upper left corner

and its revised columns. We multiply 3 factors by \( \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \) and the result is second row. We indicate a system of

given restrictions.
For discussion

- What would you do with the portion with the lines drawn through them?

**EXAMPLE 2**

In \( A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \), find the minor and cofactor of

a. \( a_{31} \)

b. \( a_{12} \)

**Solution**

a. The minor \( M_{31} \) is the minor of \( a_{31} = 7 \) appearing in row 3, column 1. It is found by deleting row 3 and column 1:

\[ M_{31} = \begin{vmatrix} 2 & 3 \\ 5 & 6 \end{vmatrix} = \frac{2 \cdot 6 - 3 \cdot 5}{-3} = -\frac{2}{3} \]

Because \( i + j \) is even (3 + 1 = 4), the cofactor of the minor \( M_{31} \) is \( M_{31} \):

\[ C_{31} = M_{31} = \begin{vmatrix} 2 & 3 \\ 5 & 6 \end{vmatrix} = 2 \cdot 6 - 3 \cdot 5 = 12 - 15 = -3 \]

b. The minor \( M_{12} \) is the minor of \( a_{12} = 2 \) appearing in row 1, column 2. It is found by deleting row 1 and column 2.

\[ M_{12} = \begin{vmatrix} 2 & 3 \\ 5 & 6 \end{vmatrix} = \frac{4 \cdot 9 - 6 \cdot 7}{-6} = -6 \]

Because \( i + j \) is odd (1 + 2 = 3), the cofactor of the minor \( M_{12} \) is \(-M_{12}\):

\[ C_{12} = -M_{12} = - \begin{vmatrix} 4 & 6 \\ 7 & 9 \end{vmatrix} = -(4 \cdot 9 - 6 \cdot 7) = -(36 - 42) = 6 \]
The determinants below are spatial problems. A blank line is needed above and below.

In print, red type is shown for parts of the problem. It is described well in the text, and is not shown in braille.

The English letter indicator is not used with any letter or combination of letters in a determinant or matrix.

\[
y = \frac{af - ec}{ad - bc}
\]

We can write the values of \( x \) and \( y \) in Equations 3 and 4 using determinants.

\[
x = \frac{e\; b}{f\; d} = \frac{ed - bf}{ad - bc} \quad \text{and} \quad y = \frac{a\; e}{c\; f} = \frac{af - ec}{ad - bc}
\]

If we compare these formulas with the original system,

\[
\begin{align*}
ax + by & = e \\
ax + dy & = f
\end{align*}
\]

we see that the denominators are the determinant of the coefficient matrix:

\[
\text{Denominator determinant} = \begin{vmatrix} a & b \\ c & d \end{vmatrix}
\]

To find the numerator determinant for \( x \), we replace the \( a \) and \( c \) in the first column of the denominator determinant with the constants \( e \) and \( f \).

To find the numerator determinant for \( y \), we replace the \( b \) and \( d \) in the second column of the denominator determinant with the constants \( e \) and \( f \).

\[
x = \frac{e\; b}{f\; d} = \frac{a\; e}{c\; f} \quad \text{and} \quad y = \frac{a\; b}{c\; d}
\]

This method of using determinants to solve systems of equations is called Cramer's rule.
We can write $y = bx + cy + d$ equivalent to $y = bx + cy + e$ using determinants.

\[
\begin{vmatrix}
1 & b & c \\
1 & b & c \\
1 & b & c \\
\end{vmatrix}
\]

If we update $y$ to be $y'$ in the original system,

\[
\begin{vmatrix}
1 & b & c \\
1 & b & c \\
1 & b & c \\
\end{vmatrix}
\]

We see that the denominators of the determinant vs coefficient matrix.
Denominator Determinant

\[ \begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} \]

To find a denominator determinant, we replace 2 by \( x \) in its first column and replace 1 by \( x \) in its second column to get:

\[ \begin{vmatrix} x & 3 \\ 2 & 2 \end{vmatrix} \]

\[ = x \cdot 2 - 3 \cdot 2 = 2x - 6 \]

\[ = 2(x - 3) \]

\[ = 2 \cdot 0 = 0 \]

\[ = 2 \cdot 0 = 0 \]
A method to use determinants to solve systems of equations is called the Scrambles rule. ...
- The example below shows a determinant that won’t fit across the line. Notice that the expression is centered above the fraction line. A blank line is required before and after a spatial arrangement (determinant), Section 183, Nemeth Code.

- I would put in a transcriber’s note before the section of the determinant that has to be split between lines. Note: The numerator and determinant of the matrix below each have been split into 2 parts.

\[
\begin{align*}
\text{Example 7} \\
\text{Use Cramer's rule to solve the system} \\
\begin{cases}
2x - y + 2z = 3 \\
x - y + z = 2 \\
x + y + 2z = 3
\end{cases}
\end{align*}
\]

\[
\text{Solution} \\
\text{Each of the values } x, y, \text{ and } z \text{ is the quotient of two } 3 \times 3 \text{ determinants. The denominator of each quotient is the determinant consisting of the nine coefficients of the variables. The numerators for } x, y, \text{ and } z \text{ are modified copies of this denominator determinant. We substitute the column of constants for the coefficients of the variable for which we are solving.}
\]

\[
\begin{align*}
\begin{vmatrix}
3 & -1 & 2 \\
2 & -1 & 1 \\
3 & 1 & 2
\end{vmatrix}
&= \begin{vmatrix}
-1 & 1 & 2 \\
1 & 2 & 3 \\
2 & 3 & 1
\end{vmatrix} + \begin{vmatrix}
2 & -1 \\
1 & 1 \\
1 & 2
\end{vmatrix}
\end{align*}
\]

\[
\begin{align*}
\begin{vmatrix}
3 & -1 & 2 \\
2 & -1 & 1 \\
1 & 1 & 2
\end{vmatrix}
&= \begin{vmatrix}
-1 & 1 & 2 \\
1 & 1 & 2 \\
1 & 1 & 1
\end{vmatrix} + \begin{vmatrix}
2 & -1 \\
1 & 1 \\
1 & 1
\end{vmatrix}
\end{align*}
\]
Example:

Use Cramer's Rule to solve the system:

\[\begin{align*}
\text{Solve for } x, y, & \text{ and } z.
\end{align*}\]